KALKER’S COEFFICIENT $c_{11}$ AND ITS INFLUENCE ON THE DAMPING AND THE RETUNING OF A MECHANICAL DRIVE TORSION SYSTEM OF A RAILWAY VEHICLE

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Introduction

- Research of electromechanical phenomena in drive systems of high-power rail vehicles
  - Electromagnetically excited torsion oscillations – identification and elimination
  - Influence of the adhesion phenomenon on torsion oscillations – identification and elimination

- Tools:
  - Basic calculation model – natural frequencies and natural modes of oscillations
  - Complex simulation model – simulation of a drive of a vehicle

Figure 2 A fully-suspended drive of a locomotive [2]
Basic mathematical model

1. Model with no wheel-rail contact implemented

\[
[J][\ddot{\varphi}] + [k][\varphi] = [M]
\]

\[
[J][\ddot{\varphi}] + [k][\varphi] = [0]
\]

- eigenvector \([\varphi_{ij}]\)
- eigenvalue vector \([\lambda_j]\)

\[
f_j = \frac{\sqrt{\lambda_j}}{2\pi}
\]
Basic mathematical model

2. Model with no wheel-rail contact implemented

Kalker’s linear theory:
\[ T_1 = c_{11} a_{el} b_{el} G s_X = C_1 s_X = k_1 s_X \]
Basic mathematical model

2. Model with no wheel-rail contact implemented

Kalker’s linear theory:
\[ T_1 = c_{11} a_{el} b_{el} G s_X = C_1 s_X = k_1 s_X \]

Figure 4 Visualization of wheel-rail forces - top view

Figure 6 Popovici’s adhesion characteristics [7]
2. Model with no wheel-rail contact implemented

Kalker’s linear theory:
\[ T_1 = c_{11} a_{el} b_{el} Gs_X = C_1 s_X = k_1 s_X \]

**Figure 4 Visualization of wheel-rail forces - top view**

**Figure 8 Torsion system scheme - fully-suspended drive**
Basic mathematical model

2. Model with no wheel-rail contact implemented

Kalker’s linear theory:

\[ T_1 = c_{11} a_e b_e G s_X = C_1 s_X = k_1 s_X \]

\[ c_{11} = 4,984 \]

Figure 8 Torsion system scheme - fully-suspended drive

Figure 9 Vehicle traction characteristic

\[ [U][\ddot{\phi}] + [b][\dot{\phi}] + [k][\phi] = [0] \]

\[ b_{W-R} = C_1 \frac{r_k^2}{\nu} \]
### Calculation results

#### Table 2 Description of natural modes [3]

<table>
<thead>
<tr>
<th>Order of natural modes</th>
<th>Respective natural frequency [Hz]</th>
<th>Dominant oscillations of a mass</th>
<th>Less significant oscillations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0</td>
<td>Own free rotation</td>
<td>-</td>
</tr>
<tr>
<td>2.</td>
<td>6</td>
<td>Wheel-set towards hollow shaft</td>
<td>-</td>
</tr>
<tr>
<td>3.</td>
<td>57</td>
<td>Wheels of wheel-set</td>
<td>-</td>
</tr>
<tr>
<td>4.</td>
<td>337</td>
<td>Wheel-set towards hollow shaft</td>
<td>Pinion towards rotor</td>
</tr>
<tr>
<td>5.</td>
<td>572</td>
<td>Pinion towards rotor</td>
<td>-</td>
</tr>
<tr>
<td>6.</td>
<td>857</td>
<td>Hollow shaft joints</td>
<td>Wheel-set towards hollow shaft Gear wheel towards hollow shaft</td>
</tr>
<tr>
<td>7.</td>
<td>2403</td>
<td>Pinion towards rotor</td>
<td>Pinion towards gear wheel</td>
</tr>
</tbody>
</table>
Calculation results

\[ c_{11} = 4,984 = C_1 \]

\[ b_{W-R} = C_1 \frac{r_k^2}{v} \]

\[ \lambda_j = -\delta_j \pm i\Omega_{dmp,j} \]

\[ f_{dmp,j} = \frac{\Omega_{dmp,j}}{2\pi} \]
Calculation results

\[ c_{11/2} = 2.492 = C_1 \]

\[ b_{W-R} = C_1 \frac{r_k^2}{v} \]

\[ \lambda_j = -\delta_j \pm i\Omega_{dmp,j} \]

\[ f_{dmp,j} = \frac{\Omega_{dmp,j}}{2\pi} \]

Figure 19 Damping as a function of velocity - \( c_{11/2} \)

Figure 20 Natural frequencies as a function of velocity - \( c_{11/2} \)
Calculation results

\[ c_{11}/4 = 1,246 = C_1 \]

\[ b_{W-R} = C_1 \frac{r_k^2}{v} \]

\[ \lambda_j = -\delta_j \pm i\Omega_{dmp,j} \]

\[ f_{dmp,j} = \frac{\Omega_{dmp,j}}{2\pi} \]

Figure 21 Damping as a function of velocity - \( c_{11}/4 \)

Figure 22 Natural frequencies as a function of velocity - \( c_{11}/4 \)
Conclusion

- The wheel-rail contact influences significantly behaviour of the torsion system – specific natural frequencies – via its damping capability:
  - Natural frequency and natural model of torsion oscillations related to oscillations of the wheel-set itself.
  - Natural frequency and natural model of torsion oscillations related to oscillations of the wheel-set towards the hollow shaft.

- The wheel-rail contact does not influence the rest of the torsion system.

The practical meaning:

- For the research oriented on a wheel-set oscillations phenomenon it SHALL BE CONSIDERED.
- For the research oriented on the torsion oscillations of the rest of traction drive components it can be neglected.
References


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