# KALKER'S COEFFICIENT $c_{11}$ AND ITS INFLUENCE ON THE DAMPING AND THE RETUNING OF A MECHANICAL DRIVE TORSION SYSTEM OF A RAILWAY VEHICLE

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**2. 6. 2021**

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# Introduction

- Research of electromechanical phenomena in drive systems of high-power rail vehicles
  - Electromagnetically excited torsion oscillations – identification and elimination
  - Influence of the adhesion phenomenon on torsion oscillations – identification and elimination

- Tools:
  - Basic calculation model – natural frequencies and natural modes of oscillations
  - Complex simulation model – simulation of a drive of a vehicle

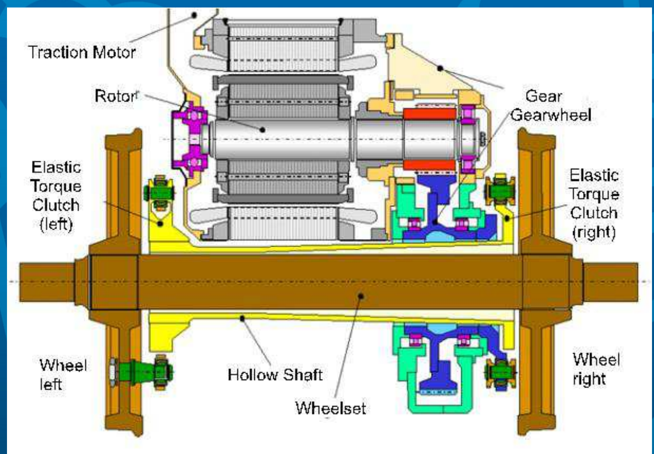



**Figure 2 A fully-suspended drive of a locomotive [2]**

# Basic mathematical model

1. Model with no wheel-rail contact implemented

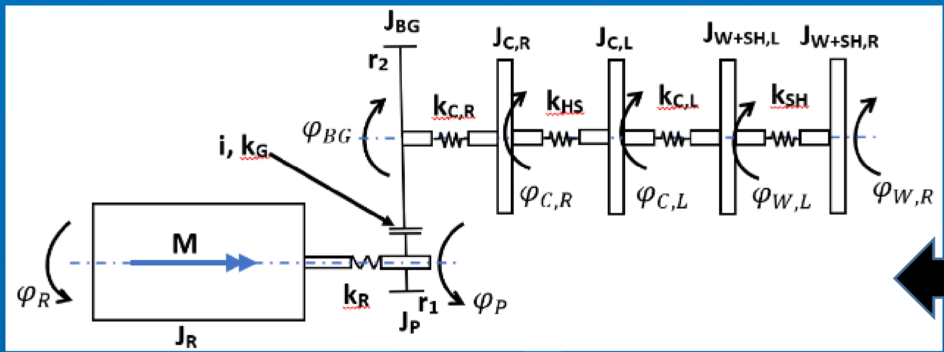



**Figure 3 Torsion system scheme - fully-suspended drive [3]**

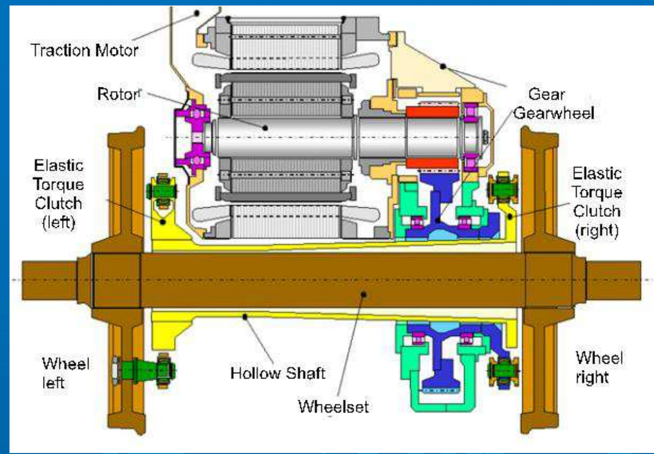



**Figure 2 A fully-suspended drive of a locomotive [2]**

$$[J][\ddot{\varphi}] + [k][\varphi] = [M]$$

$$[J][\ddot{\varphi}] + [k][\varphi] = [0] \begin{cases} \text{eigenvector } [\varphi_{i,j}] \\ \text{eigenvalue vector } [\lambda_j] \end{cases}$$

$$f_j = \frac{\sqrt{\lambda_j}}{2\pi}$$

# Basic mathematical model

2. Model with no wheel-rail contact implemented

Kalker's linear theory:

$$T_1 = c_{11} a_{el} b_{el} G s_X \ = \ C_1 s_X = k_1 s_X$$

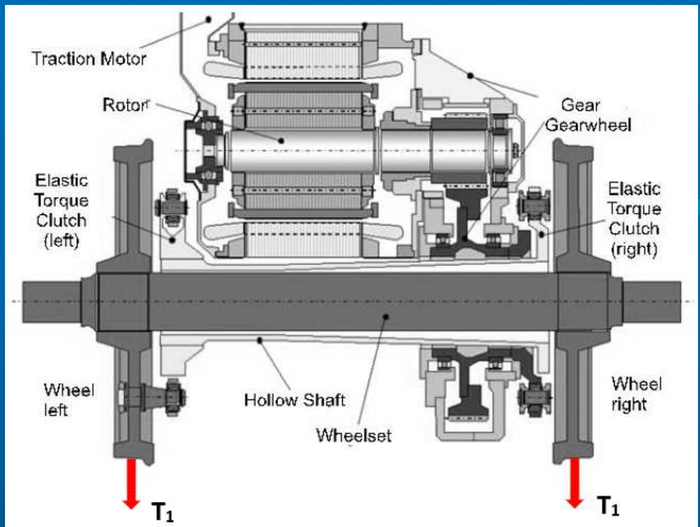



**Figure 4 Visualization of wheel-rail forces - top view**

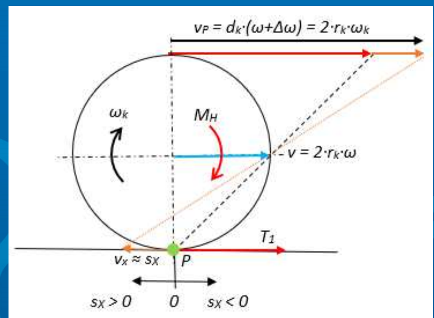



**Figure 7 Wheel slip deduction [2]**

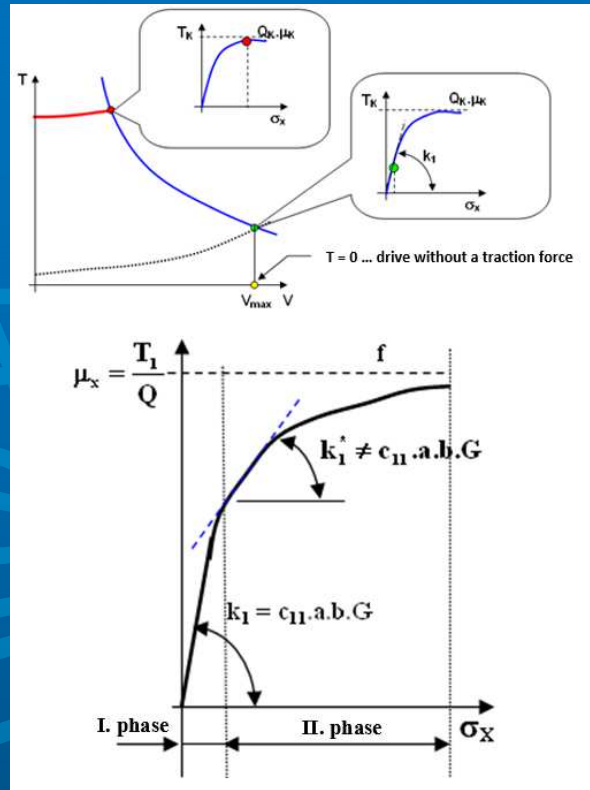



**Figure 5 Traction characteristic and adhesion characteristics of wheelset [4] [5] [6]**

# Basic mathematical model

2. Model with no wheel-rail contact implemented

Kalker's linear theory:

$$T_1 = c_{11} a_{el} b_{el} G s_X = C_1 s_X = k_1 s_X$$

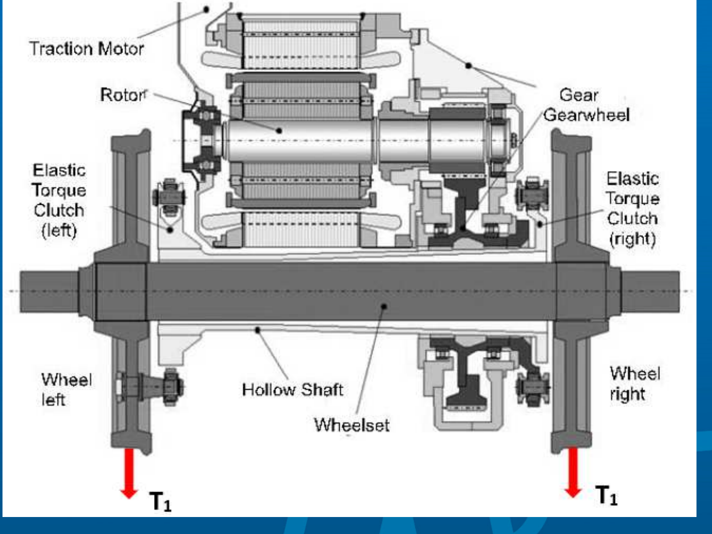



**Figure 4 Visualization of wheel-rail forces - top view**

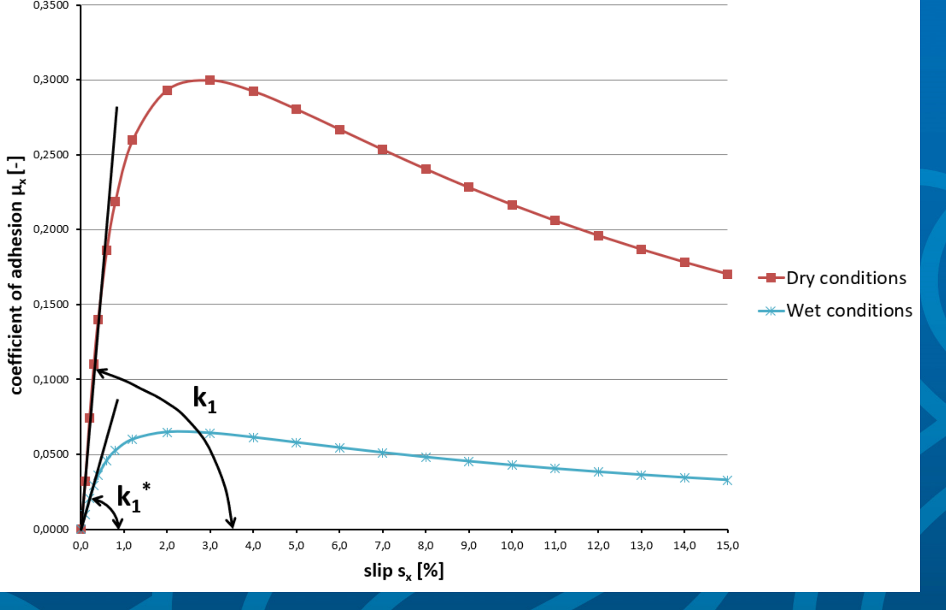



**Figure 6 Popovici's adhesion characteristics [7]**

# Basic mathematical model

2. Model with no wheel-rail contact implemented

Kalker's linear theory:

$$T_1 = c_{11} a_{el} b_{el} G s_X = C_1 s_X = k_1 s_X$$

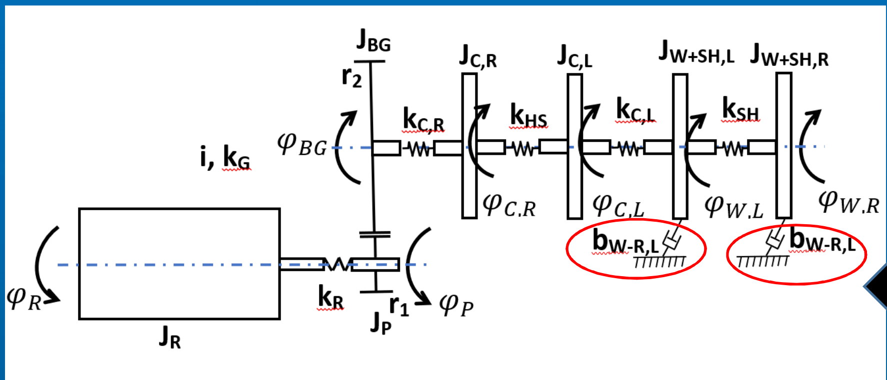



Figure 8 Torsion system scheme - fully-suspended drive

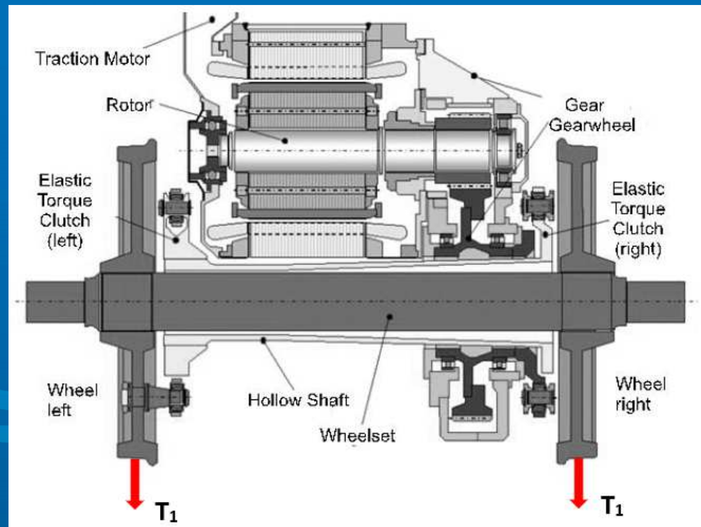



Figure 4 Visualization of wheel-rail forces - top view

$$b_{W-R} = C_1 \frac{r_k^2}{v}$$

$$[J][\ddot{\varphi}] + [b][\dot{\varphi}] + [k][\varphi] = [0] \begin{cases} \text{eigenvector } [\varphi_{i,j}] \\ \text{eigenvalue vector } [\lambda_j = -\delta_j \pm i\Omega_{dmp,j}] \; f_{dmp,j} = \dfrac{\Omega_{dmp,j}}{2\pi} \end{cases}$$

# Basic mathematical model

2. Model with no wheel-rail contact implemented

Kalker's linear theory:

$$T_1 = c_{11} a_{el} b_{el} G s_X = C_1 s_X = k_1 s_X$$

$c_{11} = 4{,}984$

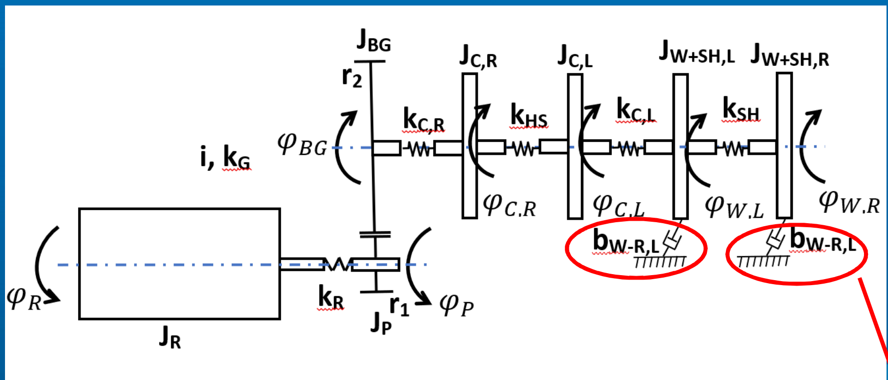



**Figure 8 Torsion system scheme - fully-suspended drive**

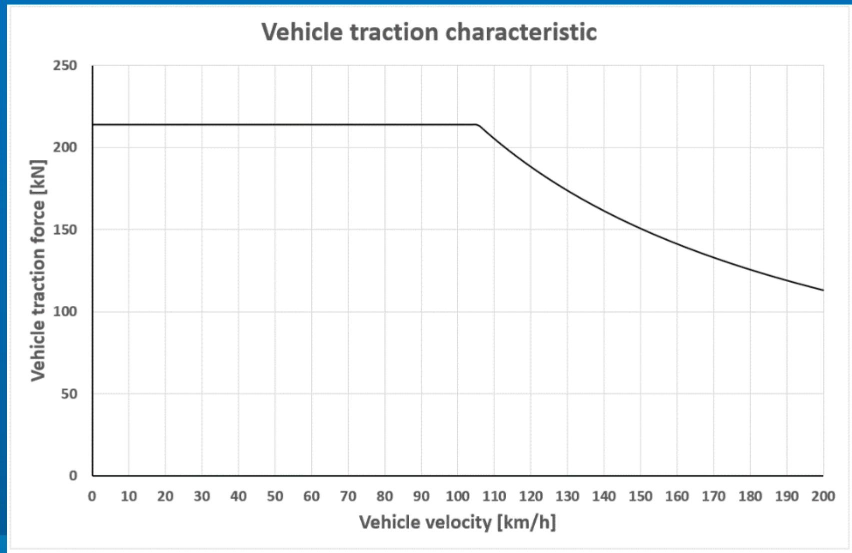



**Figure 9 Vehicle traction characteristic**

$$[J][\ddot{\varphi}] + [b][\dot{\varphi}] + [k][\varphi] = [0]$$

$$b_{W-R} = C_1 \frac{r_k^2}{v}$$

# Calculation results

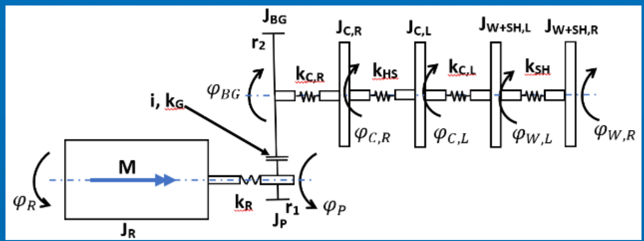


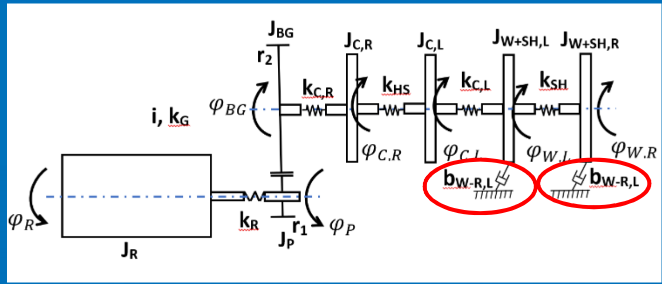

**Figure 3 Torsion system scheme – fully-suspended drive [3]**

$$f_j = \frac{\sqrt{\lambda_j}}{2\pi}$$

**Table 2 Description of natural modes [3]**

| Order of natural modes | Respective natural frequency [Hz] | Dominant oscillations of a mass | Less significant oscillations |
|---|---|---|---|
| 1. | 0 | Own free rotation | - |
| 2. | 6 | Wheel-set towards hollow shaft | - |
| **3.** | **57** | **Wheels of wheel-set** | **-** |
| 4. | 337 | Wheel-set towards hollow shaft | Pinion towards rotor |
| 5. | 572 | Pinion towards rotor | - |
| 6. | 857 | Hollow shaft joints | Wheel-set towards hollow shaft<br>Gear wheel towards hollow shaft |
| 7. | 2403 | Pinion towards rotor<br>Pinion towards gear wheel | - |

# Calculation results




$$c_{11} = 4{,}984 = C_1 \longrightarrow b_{W-R} = C_1 \frac{r_k^2}{v}$$

$$\lambda_j = -\delta_j \pm i\Omega_{dmp,j} \longrightarrow f_{dmp,j} = \frac{\Omega_{dmp,j}}{2\pi}$$

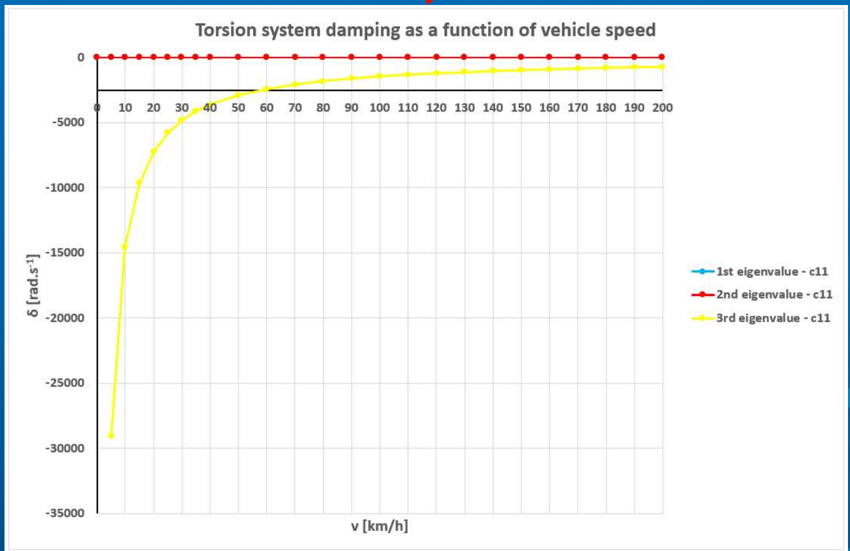



**Figure 17 Damping as a function of velocity - $c_{11}$**

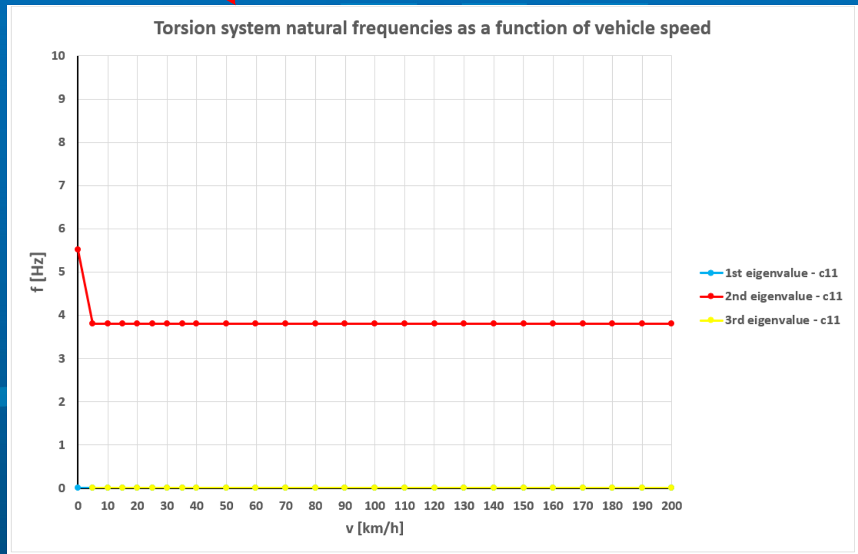
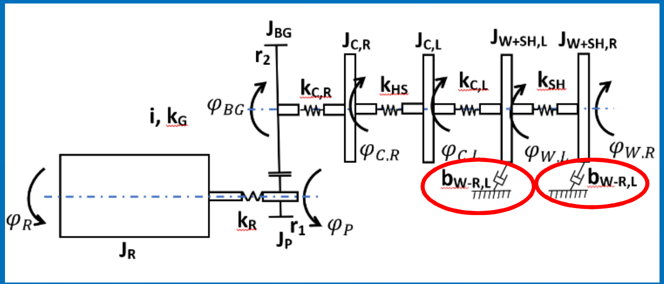



**Figure 18 Natural frequencies as a function of velocity - $c_{11}$**

# Calculation results

$$c_{11}/2 = 2{,}492 = C_1 \longrightarrow b_{W-R} = C_1 \frac{r_k^2}{v}$$




$$\lambda_j = -\delta_j \pm i\Omega_{dmp,j} \longrightarrow f_{dmp,j} = \frac{\Omega_{dmp,j}}{2\pi}$$

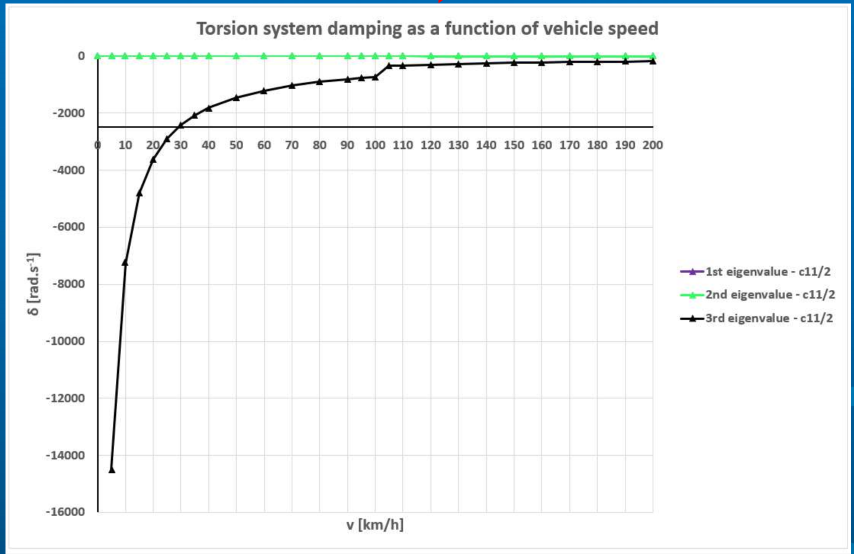



**Figure 19 Damping as a function of velocity - $c_{11}/2$**

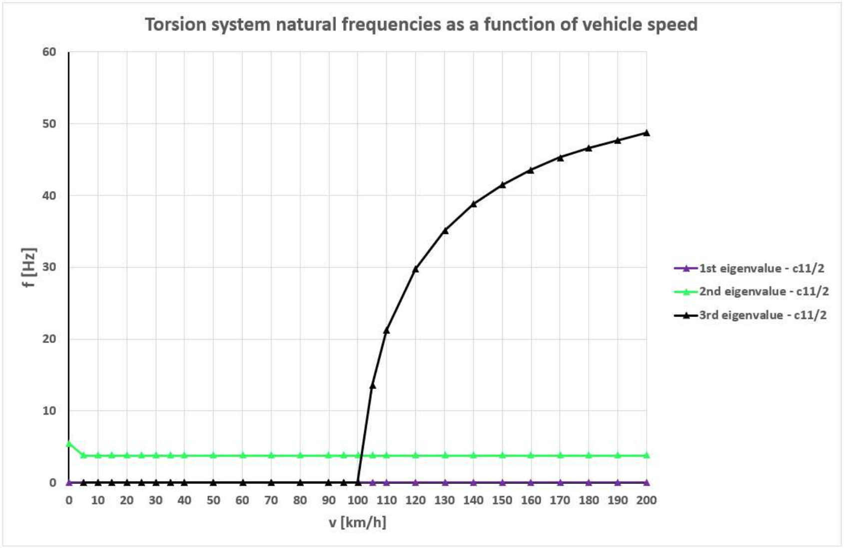
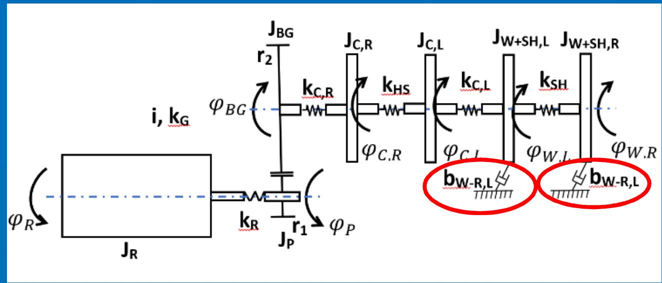



**Figure 20 Natural frequencies as a function of velocity - $c_{11}/2$**

# Calculation results

$$c_{11}/4 = 1{,}246 = C_1 \longrightarrow b_{W-R} = C_1 \frac{r_k^2}{v}$$




$$\lambda_j = -\delta_j \pm i\Omega_{dmp,j} \longrightarrow f_{dmp,j} = \frac{\Omega_{dmp,j}}{2\pi}$$

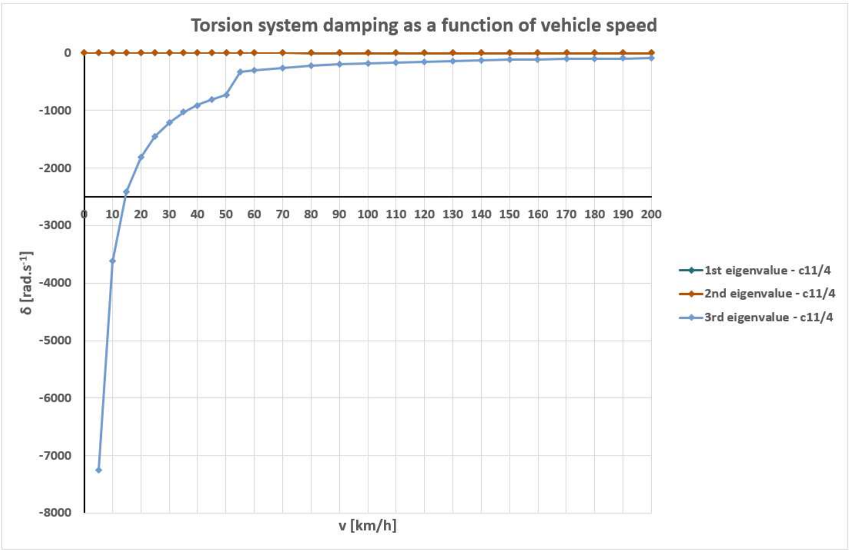



Figure 21 Damping as a function of velocity - $c_{11}/4$

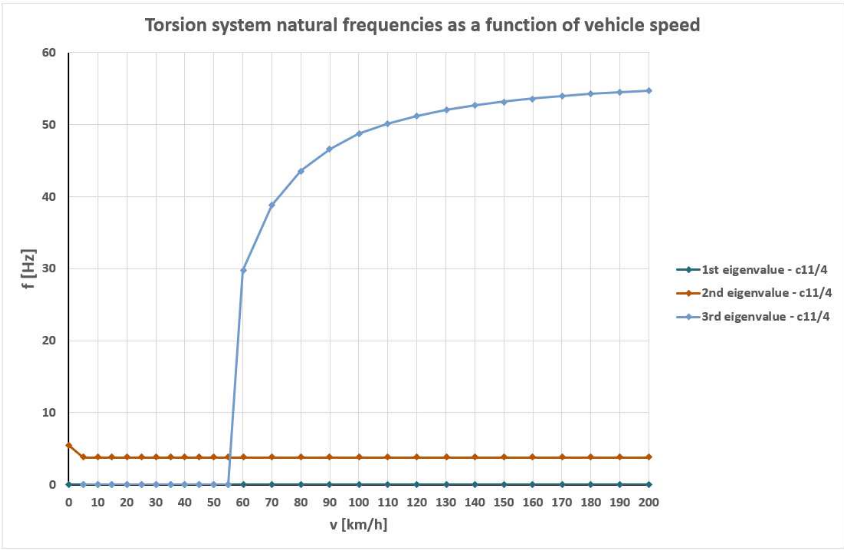



Figure 22 Natural frequencies as a function of velocity - $c_{11}/4$

# Conclusion

- The wheel-rail contact influences significantly behaviour of the torsion system – specific natural frequencies – via its damping capability:
  - Natural frequency and natural model of torsion oscillations related to oscillations of the wheel-set itself.
  - Natural frequency and natural model of torsion oscillations related to oscillations of the wheel-set towards the hollow shaft.

- The wheel-rail contact does not influnce the rest of the torsion system.

The practical meaning:

- For the research oriented on a wheel-set oscillations phenomenon It SHALL BE CONSIDERED.

- For the research oriented on the torsion oscillations of the rest of traction drive components it can be neglected.

# References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## Acknowledgement

This research has been realized using the support of The Technology Agency of the Czech Republic, programme National Competence Centres, project # TN01000026 Josef Bozek National Center of Competence for Surface Transport Vehicles This support is gratefully acknowledged.